

# $\sqrt{CCML}$ Video Contest – Meet 3 2021-2022

## Guidelines

- Students from each half of your team (freshman/sophomore or junior/senior) from your school may submit up to two videos on the given problem. Each video submitted must be produced by different students, but must all be from the appropriate grade band. If your school decides to submit two f/s videos, there should be different students in each video.
- Each video should be no more than SIX minutes in length. Note that this does not mean that you have to fill the entire six minutes.
- The problems are to be solved and the videos produced by student groups. The bulk of the work should be done by students. A parent or teacher holding a camera is fine, but solving a problem for the students is not.
- Videos must be produced by a group of at least two students, and at most five students. Each participating student's contribution should be made evident either from an appearance in the video or a credit at the beginning or end of the video. Indicate names of all students involved (maximum of 5) in credits or introductions at the beginning or end of the video.
- The top f/s video and j/s video from your school will earn points for your overall team score according to the attached rubric.
- Creative solutions and presentations are encouraged, but correct math is paramount. Please make the focus of your video the mathematics. If you have a creative context, great, but it should not be the focus of your video. Soundtracks should not distract or interfere with the explanation of the solution.

## Submission

- Coaches should ensure that no more than two videos per grade band are submitted.
- **Make sure that videos are viewable by anyone with the link!**
- Coaches should upload videos to Google drive and share access with Michael Caines (macaines@cps.edu). Please use the following naming conventions for the videos: **school\_level\_teamnumber\_contestnumber\_year**. For example, a submission for CCML 3 for a f/s team from Kelly in the 2015–2016 school year should be named as follows, **kelly\_fs\_team1\_contest3\_1516**. A submission from a j/s team from Lakeview should be named **lakeview\_js\_team1\_contest3\_1516**
- **All submissions must be shared by 5pm on Tuesday, January 18, 2022.**

Please direct any questions about the contest to Michael Caines (macaines@cps.edu). Coaches who are interested in helping judge the submissions should email Michael Caines by the submission deadline.

## Problems:

- **Freshman/Sophomore Problems**

Consider the set  $S$  of points  $(x, y)$  where  $x$  and  $y$  are integers chosen, with replacement, from the interval  $[-2, 2]$ .

(a) What is the greatest possible area of a right triangle that can be made with vertices in  $S$ ?

(b) If points  $A$ ,  $B$ , and  $C$  are chosen from  $S$ , without replacement, what is the probability that

$$AB < AC + BC \text{ and } AC < AB + BC \text{ and } BC < AB + AC?$$

(c) If two elements of  $S$  are chosen, without replacement, what is the probability that the slope of the line containing those points is an integer?

- **Junior/Senior Problems:**

The *dot product* of vectors  $\mathbf{v} = \langle v_x, v_y \rangle$  and  $\mathbf{w} = \langle w_x, w_y \rangle$  is denoted  $\mathbf{v} \cdot \mathbf{w}$  and is computed as  $v_x w_x + v_y w_y$ .

(a) Given vectors  $\mathbf{a} = \langle c, 2c \rangle$  and  $\mathbf{b} = \langle 4, c \rangle$ , solve for  $c$  such that  $\mathbf{a} \cdot \mathbf{b} = 0$ .

(b) By assuming that  $\mathbf{v}^2 = \mathbf{v} \cdot \mathbf{v}$ , we can extend the notion of standard deviation to a set of vectors. Given that  $x$  is a real number, determine the value of  $x$  that minimizes the standard deviation of the following set:

$$\{\langle -1, 0 \rangle, \langle 1, 1 \rangle, \langle 1, 0 \rangle, \langle 0, -1 \rangle, \langle x, 1 \rangle\}$$

(c) Consider the set  $S$  of vectors  $\langle x, y \rangle$  where  $x$  and  $y$  are integers chosen, with replacement, from the interval  $[-3, 3]$ . What is the probability that, in a given permutation of the elements of  $S$ , the lengths of the vectors are nonincreasing?

## CCML Video Contest Rubric

Team Name: \_\_\_\_\_ Contest: \_\_\_\_\_ Year: \_\_\_\_\_

<b>Part (a)</b>	<b>0</b>	<b>1</b>		<b>2</b>
	<ul style="list-style-type: none"> <li>No attempt is made, or the work contains profound errors.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains some good work, but also nontrivial errors.</li> <li>Explanation of work is unclear.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains only trivial errors or no errors.</li> <li>Explanation of work is clear.</li> </ul>	
<b>Part (b)</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
	<ul style="list-style-type: none"> <li>No attempt is made, or the work contains profound errors.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains some good work, but also multiple nontrivial errors.</li> <li>Explanation of work is unclear.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains no more than one nontrivial error.</li> <li>Explanation of work is generally clear.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains only trivial errors or no errors.</li> <li>Explanation of work is clear.</li> </ul>
<b>Part (c)</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
	<ul style="list-style-type: none"> <li>No attempt is made, or the work contains profound errors.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains some good work, but also multiple nontrivial errors.</li> <li>Explanation of work is unclear.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains no more than one nontrivial error.</li> <li>Explanation of work is generally clear.</li> </ul>	<ul style="list-style-type: none"> <li>Problem contains only trivial errors or no errors.</li> <li>Explanation of work is clear.</li> </ul>
<b>Presentation</b>	<b>0</b>		<b>1</b>	<b>2</b>
	<ul style="list-style-type: none"> <li>Images are sloppy or out of focus.</li> <li>Audio is difficult to hear.</li> </ul>		<ul style="list-style-type: none"> <li>Audio/video are clear.</li> <li>Presentation is organized well</li> </ul>	<ul style="list-style-type: none"> <li>Presentation is truly creative and engaging.</li> </ul>

Score: \_\_\_\_\_ / 10

Notes: