# $\sqrt{CCML}$ Video Contest – Meet 5 2019-2020

### Guidelines

- Students from each half of your team (freshman/sophomore or junior/senior) from your school may submit up to two videos on the given problem. Each video submitted must be produced by different students, but must all be from the appropriate grade band. If your school decides to submit two f/s videos, there should be different students in each video.
- Each video should be no more than SIX minutes in length. Note that this does not mean that you have to fill the entire six minutes.
- The problems are to be solved and the videos produced by student groups. The bulk of the work should be done by students. A parent or teacher holding a camera is fine, but solving a problem for the students is not.
- Videos must be produced by a group of at least two students, and at most five students. Each participating student's contribution should be made evident either from an appearance in the video or a credit at the beginning or end of the video. Indicate names of all students involved (maximum of 5) in credits or introductions at the beginning or end of the video.
- The top f/s video and j/s video from your school with earn points for your overall team score according to the attached rubric.
- Creative solutions and presentations are encouraged, but correct math is paramount. Please make the focus of your video the mathematics. If you have a creative context, great, but it should not be the focus of your video. Soundtracks should not distract or interfere with the explanation of the solution.

#### Submission

- Coaches should ensure that no more than two videos per grade band are submitted.
- Coaches should upload videos to Google drive and share access with Michael Caines (macaines@cps.edu). Please use the following naming conventions for the videos: school\_level\_teamnumber\_contestnumber\_year. For example, a submission for CCML 3 for a f/s team from Kelly in the 2015–2016 school year should be named as follows, kelly\_fs\_team1\_contest3\_1516. A submission from a j/s team from Lakeview should be named lakeview\_js\_team1\_contest3\_1516
- All submissions must be shared by 5pm on Tuesday, March 31, 2020.

Please direct any questions about the contest to Michael Caines (macaines@cps.edu). Coaches who are interested in helping judge the submissions should email Michael Caines by the submission deadline.

#### **Problems:**

#### • Freshman/Sophomore Problems:

(a) A *fixed point* of a function is an input such that the output equals the input. Determine all fixed points of the function  $k(x) = x^3$ .

(b) Let  $f^n(x)$  denote the result of applying function f to x n times. That is,  $f^2(x) = f(f(x))$ ,  $f^3(x) = f(f(f(x)))$ , etc.

If  $g(x) = \frac{x+1}{x-1}$  and  $h(x) = \frac{x-1}{x+1}$ , compute  $g^{2020}(2020) - h^{2020}(2020)$ .

(c) Let g(x) and h(x) be defined as in the previous problem. If *a* and *b* are chosen, with replacement, from the set {1, 2, 3, 4}, what is the probability that  $g^{a}(x) = h^{b}(x)$  has real solutions for *x*?

#### • Junior/Senior Problems:

(a) Line *m* is described parametrically by x = 2, y = 1, z = t, for  $t \in \mathbb{R}$ . Line *n* is described by x = w, y = 1 - w, z = 0 for  $w \in \mathbb{R}$ . Determine the distance between lines *m* and *n*.

(b) Consider the set of points (x, y, z) that satisfies  $-5 \le x \le 5$  and  $-5 \le y \le 5$  and  $-5 \le z \le 5$ . If a point is randomly chosen from the interior of this region, what is the probability that it is closer to the point (1,0,0) than it is to the *y*-axis?

(c) If a point is chosen at random from the interior of a cube, what is the probability that it is closer to the center of the cube than it is to any vertex?

## **CCML Video Contest Rubric**

Team Name: \_\_\_\_\_ Contest: \_\_\_\_ Year: \_\_\_\_

	0		1			2		
Part (a)	• No attempt is made, or the work contains profound errors.		• Problem contains some good work, but also nontrivial errors.			<ul><li>Problem contains only trivial errors or no errors.</li><li>Explanation of work is clear.</li></ul>		
	• Explanation of work is unclear.							
	0	1		2		3		
Part (b)	• No attempt is made, or the work contains profound errors.	<ul> <li>Problem contains some good work, but also multiple nontrivial errors.</li> <li>Explanation of work is unclear.</li> </ul>		• •	more than one nontrivial error.		•	Problem contains only trivial errors or no errors. Explanation of work is clear.
	0	1		2			3	
Part (c)	• No attempt is made, or the work contains profound	• Problem contains some good work, but also multiple			• Problem contains no more than one nontrivial error.		•	Problem contains only trivial errors or no errors.
	errors.	<ul> <li>nontrivial errors.</li> <li>Explanation of work is unclear.</li> </ul>		•	• Explanation of work is generally clear.		•	Explanation of work is clear.
Presentation	0		1	1		2		
	<ul><li>Images are sloppy or out of focus.</li><li>Audio is difficult to hear.</li></ul>		<ul><li>Audio/video are clear.</li><li>Presentation is organized well</li></ul>			• Presentation is truly creative and engaging.		

Score: \_\_\_\_\_ / 10

Notes: